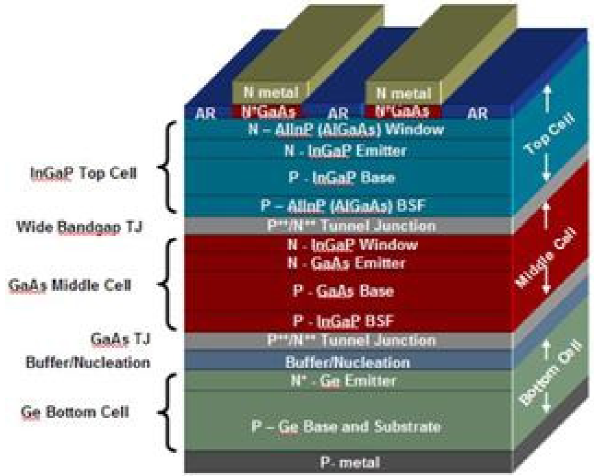
Lecture 9 – 12/11/2025

The *p-n* junction

- With a concentration gradient
- Out of equilibrium
- *I-V* characteristic



Summary Lecture 8

1 Types of Current in a p–n Junction

Drift Current

Current driven by an external electric field.

$$J_{drift} = en\mu_n E + ep\mu_p E$$

The total drift current is contributed by both:

1. Electrons moving in the **opposite** direction to the electric field
2. Holes moving in the **same** direction as the electric field

Diffusion Current

Current caused by a concentration gradient.
Carriers move from regions of high concentration to low concentration:

$$J_{diffusion} = eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}$$

Einstein Relation

$$\boxed{\frac{D}{\mu} = \frac{k_B T}{e}}$$

2. Thermal Equilibrium

At equilibrium, the net electron current is zero:

$$J_{n,net} = en\mu_n E + eD_n \frac{dn}{dx} = 0$$

The electron concentration is given by:

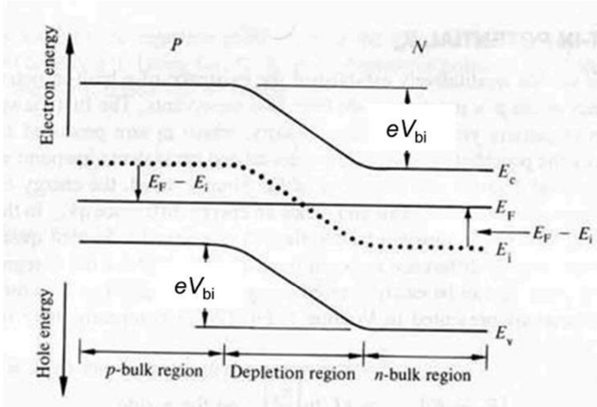
$$n = N_c \exp\left(-\frac{E_c - E_F}{k_B T}\right)$$

By substituting (n) into the first equation, we find that:

$$\frac{dE_F}{dx} = 0 \quad \Rightarrow \quad E_F = \text{constant}$$

Conclusion:
At thermal equilibrium, the Fermi level is spatially constant.

3. Built-in Potential



From the energy band diagram, we have:

$$eV_{bi} = (E_i - E_F)_p + (E_F - E_i)_n$$

where
(E_i) — intrinsic Fermi level
(E_F) — equilibrium Fermi level of the p–n junction

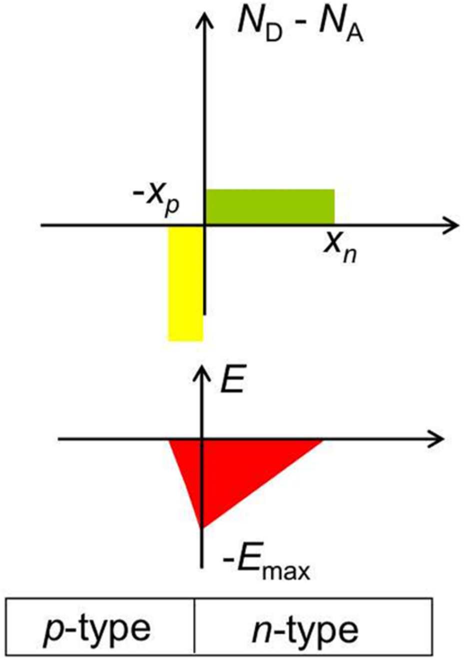
Therefore, the built-in potential is given by:

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_C N_V}{N_A N_D}\right)$$

This represents the built-in potential barrier of the p–n junction.

4. Potential and Field Distribution in the Space-Charge Region

Summary Lecture 8



The one-dimensional Poisson's equation is:

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

At equilibrium, the charge density is given by:

$$\rho(x) = \begin{cases} -eN_A, & -x_p \le x < 0 \\ +eN_D, & 0 < x \le x_n \\ 0, & \text{otherwise} \end{cases}$$

Solving Poisson's equation under this charge distribution and the boundary condition (E = 0) in the p-bulk and n-bulk regions gives:

$$E(x) = -\frac{eN_A}{\varepsilon}(x + x_p), \quad -x_p \le x \le 0$$

$$E(x) = \frac{eN_D}{\varepsilon}(x - x_n), \quad 0 \le x \le x_n$$

At the junction interface (x = 0), the electric field reaches its maximum magnitude:

$$|E_{\max}| = e\frac{N_A x_p}{\varepsilon} = e\frac{N_D x_n}{\varepsilon}$$

We define the total width of the space-charge region as:

$$W = x_p + x_n$$

Charge Neutrality

The total charge must be zero:

$$-eN_A x_p + eN_D x_n = 0 \quad \Rightarrow \quad x_p N_A = x_n N_D$$

Built-in Potential

The built-in potential is obtained by integrating the electric field:

$$V_{bi} = -\int_{-x_p}^{x_n} E(x)\,dx = \frac{eN_A x_p^2}{2\varepsilon} + \frac{eN_D x_n^2}{2\varepsilon} = \frac{1}{2}E_{\max}W$$

Using the charge neutrality condition, the total depletion width is:

$$W = \sqrt{\frac{2\varepsilon V_{bi}}{e}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

The depletion widths on each side are:

$$x_p = \frac{N_D}{N_A + N_D}W, \quad x_n = \frac{N_A}{N_A + N_D}W$$

In the special case where (N_D \gg N_A):

$$W = \sqrt{\frac{2\varepsilon V_{bi}}{eN_A}}, \quad x_p = W$$

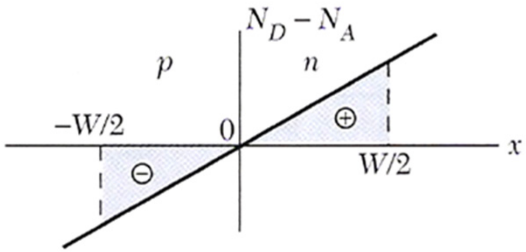
p-n junction with a gradient

- **The interface between the *n*-type and *p*-type doped layers is no longer abrupt: case of a linearly graded junction**

Density of charges given by $\rho = q\alpha x$ with α the impurity gradient (usually expressed in cm^{-4})

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q\alpha}{\varepsilon}x \qquad -\frac{W}{2} \leq x \leq \frac{W}{2}$$

Integrate between $\pm W/2$ where the electric field becomes equal to zero (space charge boundaries)



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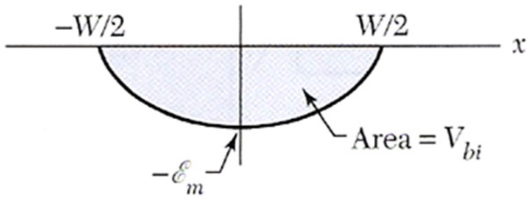
Impurity distribution



Electric field distribution

p-n junction with a gradient

The built-in potential is then

$$V_{bi} = \frac{e\alpha W^3}{12\varepsilon} \text{ and thus } W = \left(\frac{12\varepsilon V_{bi}}{e\alpha}\right)^{1/3}$$

The built-in potential can also be expressed using a form similar to that of an abrupt junction:

$$V_{bi} = \frac{k_B T}{e}\ln\left(\frac{(\alpha W/2)(\alpha W/2)}{n_i^2}\right) = \frac{2k_B T}{e}\ln\left(\frac{\alpha W}{2n_i}\right)$$

To be verified @ home!

Numerical techniques lead to:

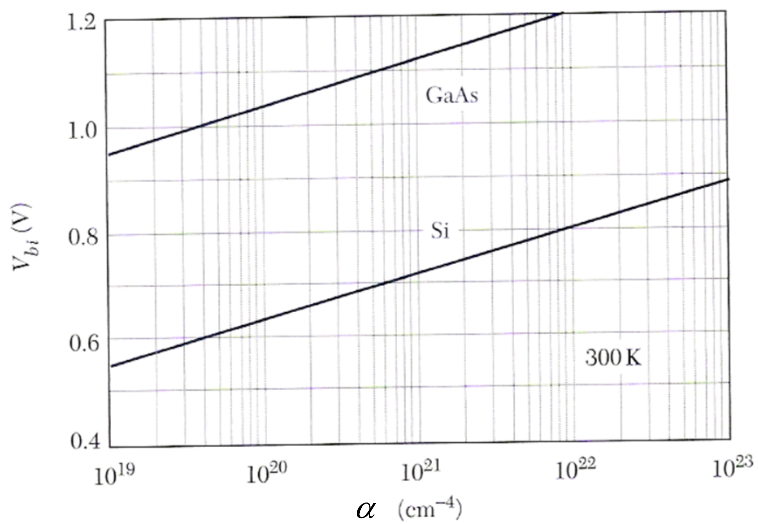
$$V_{bi} = \frac{2k_B T}{3e}\ln\left(\frac{\alpha^2 \varepsilon k_B T}{8e^2 n_i^3}\right)$$, for which V_{bi} is 0.05 to 0.1 V smaller than with the previous equation

Expression that only depends on known parameters

V_{bi} depends on the impurity gradient α

The smaller the gradient, the lower V_{bi}

W will vary as $(V_{bi} - V)^{1/3}$ whereas in an abrupt junction it varies as $(V_{bi} - V)^{1/2}$



Characteristics of the *p-n* junction at equilibrium

- Space charge
- Uncompensated fixed charges due to depleted impurities
- No net current (drift and diffusion currents compensate themselves)

The *p-n* junction out of equilibrium

p-*n* junction out of equilibrium

Out of equilibrium means that the junction is polarized (external applied bias *V*)

Then, the expression of *W* as a function of *V* is still valid provided V_{bi} is replaced by $V_{bi} - V$

$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)V_{bi}}$$

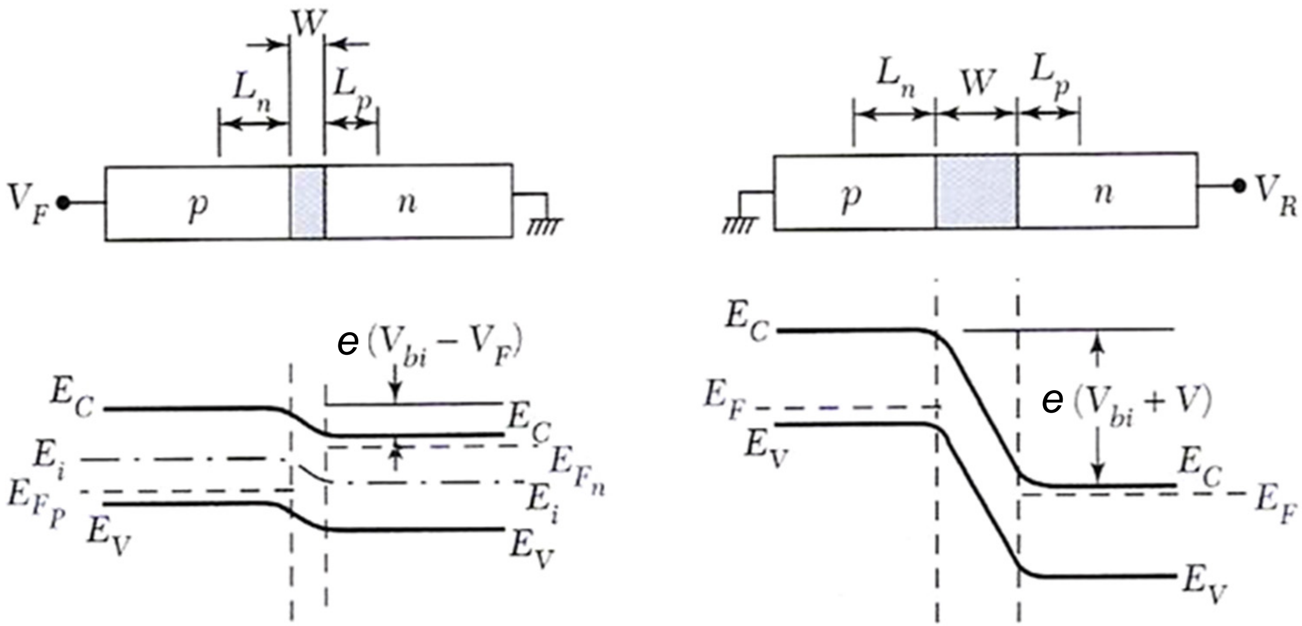
$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V)}$$

Abrupt *p*-*n* junction case!

p-*n* junction out of equilibrium

- Forward bias V_F: $V_{bi} - V_F$ thus W ⇘
- Reverse bias $-V_R$: $V_{bi} + V_R$ thus W ⇗

$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)\left(V_{bi} - V\right)}$$



E_{F_n} and E_{F_p} are the quasi-Fermi levels

Notion of quasi-Fermi levels

System driven out of equilibrium

- External generation process of electrons and holes $\Rightarrow$ generation rates G_n and G_p
- Recombination process with time constants τ_n and τ_p leading to steady-state electron and hole populations given by the equality between generation and recombination processes:

$$G_n = \frac{\Delta n}{\tau_n} \text{ and } G_p = \frac{\Delta p}{\tau_p}$$

See Lecture 7

It is assumed that the generated carriers (either by photon absorption, electrical injection, etc.) are present in densities exceeding by far thermal densities, i.e., $n, p >> n_0$ and p_0

Shockley introduced the concept which consists in considering that those free carrier populations can still be described by so-called *quasi-Fermi levels*

Notion of quasi-Fermi levels

If the system is non-degenerate, we get:

$$E_{F_n} = E_C - k_B T \ln\left(\frac{N_C}{n}\right) \text{ with } n = n_0 + G_n \tau_n \leftarrow \Delta n$$

$$E_{F_p} = E_V + k_B T \ln\left(\frac{N_V}{p}\right) \text{ with } p = p_0 + G_p \tau_p \leftarrow \Delta p$$

If the system is degenerate, we get:

$$E_{F_n} = E_C + \frac{\hbar^2}{2m_C^*}\left(3\pi^2 n\right)^{2/3}$$

$$E_{F_p} = E_V - \frac{\hbar^2}{2m_V^*}\left(3\pi^2 p\right)^{2/3}$$

As it is assumed that $G_n\tau_n >> n_0$ and $G_p\tau_p >> p_0$, quasi-Fermi levels do not coincide with each other and are even repelled from the equilibrium Fermi level position E_F by the amount:

$$\left(E_{F_n} - E_F\right) - \left(E_{F_p} - E_F\right) \approx k_B T \ln\left[\left(\frac{G_n\tau_n}{n_0}\right) / \left(\frac{G_p\tau_p}{p_0}\right)\right]$$

Non-degenerate semiconductor case

p-n junction out of equilibrium

Assumptions made for the derivation of the ideal current-voltage characteristic

1. Abrupt depletion layer, outside the depletion region the semiconductor is assumed to be neutral

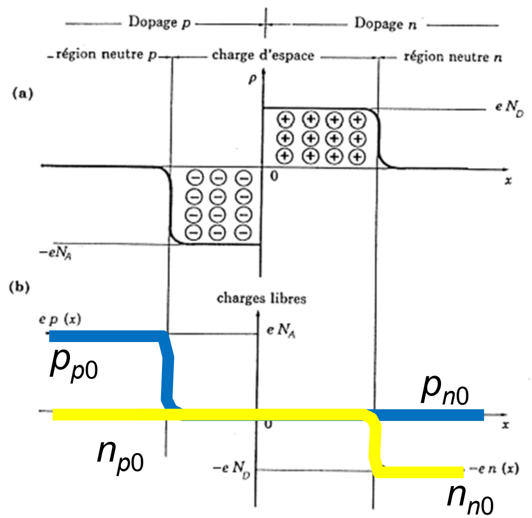
2. Carrier densities at the boundaries are related to the electrostatic potential difference across the junction

3. System operated under low-injection condition, i.e., injected minority carrier densities are small vs. majority carrier densities (i.e., $n_n \approx n_{n0}$ and $p_p \approx p_{p0}$) ⚠ $\Rightarrow$ We do not rely on the conditions of slide #11!

4. Neither generation nor recombination current exists in the depletion region, and the electron and hole currents are constant throughout the depletion region (i.e., all currents come from the neutral regions)

p-n junction out of equilibrium

$V = 0$



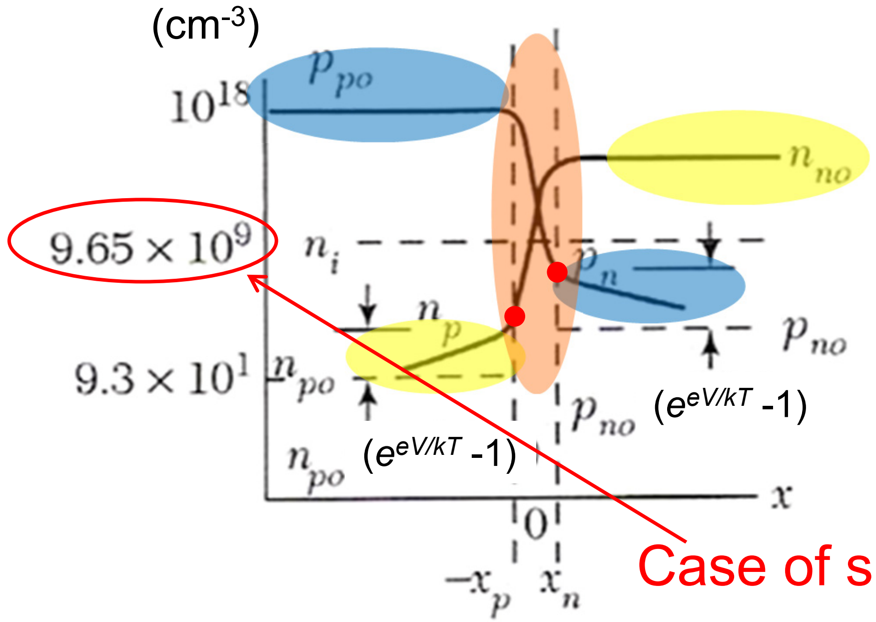
***n*-type:**

$n_{n0} = N_D$ and $p_{n0} = n_i^2/N_D$

***p*-type:**

$p_{p0} = N_A$ and $n_{p0} = n_i^2/N_A$

$V > 0$



n_p and p_n are the concentrations of electrons and holes, respectively, at the space charge boundaries

I-*V* characteristic

At equilibrium in the *n*-type and *p*-type doped layers, we have:

$$n = N_c e^{-(E_C - E_F)/k_B T}$$

$$p = N_v e^{-(E_F - E_V)/k_B T}$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_V N_C}{N_A N_D}\right) = E_g - k_B T \ln\left(\frac{np}{N_A N_D} e^{(E_C - E_F)/k_B T} e^{(E_F - E_V)/k_B T}\right) = k_B T \ln\left(\frac{N_A N_D}{np}\right)$$

$$= k_B T \ln\left(\frac{n_{n0} p_{p0}}{n_i^2}\right) = k_B T \ln\left(\frac{n_{n0}}{n_{p0}}\right) \quad \text{with } n_{p0} p_{p0} = n_i^2 \qquad n_{n0} = N_D \text{ and } p_{p0} = N_A$$

We can deduce

$$n_{p0} = n_{n0} \exp(-eV_{bi}/k_B T)$$

and

$$n_{n0} = n_{p0} \exp(eV_{bi}/k_B T)$$

At thermal equilibrium

Electron and hole concentrations at both sides of the space charge region are a function of V_{bi}

I-V characteristic

$n_{n0} = n_{p0} \exp(eV_{bi}/k_B T)$
$p_{p0} = p_{n0} \exp(eV_{bi}/k_B T)$

These relations are still valid when the junction is biased

Polarization (forward: $V > 0$, reverse: $V < 0$)

$$\boldsymbol{n_n = n_p \exp[e(V_{bi}-V)/k_B T]}$$

Assumption #2 of slide #12

with n_n and n_p the **concentrations at the space charge region boundaries**

We consider the case of the low injection regime, i.e., $n_n \approx n_{n0}$

$$n_{n0} \approx n_n \Rightarrow n_{p0}\exp(eV_{bi}/k_B T) \approx n_p \exp[e(V_{bi}-V)/k_B T]$$

and finally $\boldsymbol{n_p \approx n_{p0} \exp(eV/k_B T)}$

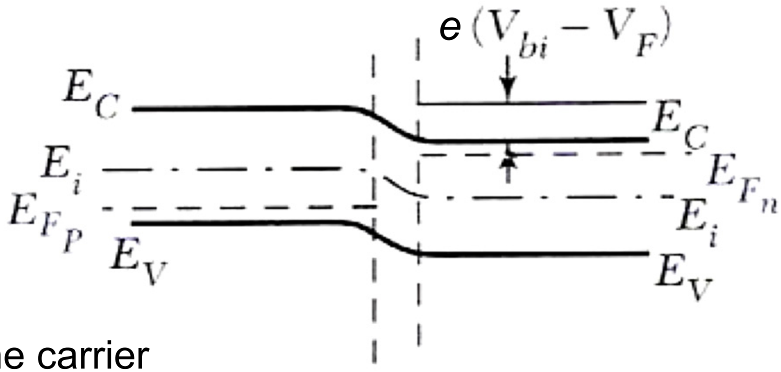
Excess or deficit of minority free carriers vs thermal equilibrium

or $n_p - n_{p0} \approx n_{p0}[\exp(eV/k_B T)-1]$ on the p-type side at $x = -x_p$

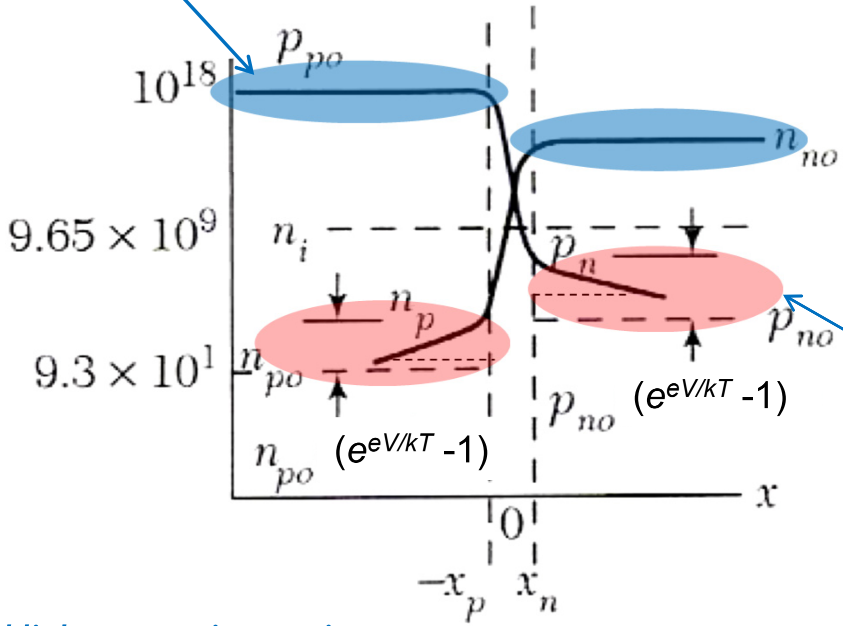
Similarly, we have $p_n - p_{n0} = p_{n0}[\exp(eV/k_B T)-1]$ on the n-type side at $x = x_n$

Shockley's relations

I-V characteristic



We have expressed the carrier concentrations at both sides of the space charge region



We have to describe the evolution of the minority carrier concentration far from the junction (when going back to equilibrium)

I-V characteristic

Minority carrier concentrations in the neutral *n*- and *p*-type doped layers

The carriers come from the *n*- and *p*-type doped layers

Currents:

Rate equation: $$\frac{dn}{dt} = \frac{1}{e}\nabla \mathbf{J_n} + \left(G_n - R_n\right)$$

G_n: generation rate

R_n: recombination rate

with $$\mathbf{J_n} = e\left(\mu_n n\mathbf{E} + D_n \vec{\nabla} n\right)$$

Reminder

$G_n - R_n$ is given by:

$$G_n - R_n = -\frac{n_p - n_{p_0}}{\tau_n}$$

Out of equilibrium population ($n_p - n_{p_0}$)

Lifetime (relaxation time) (τ_n)

Cf. Lecture 7

I-V characteristic

Steady-state regime (with no electric field in the neutral part of the *p*- and *n*-type doped regions)

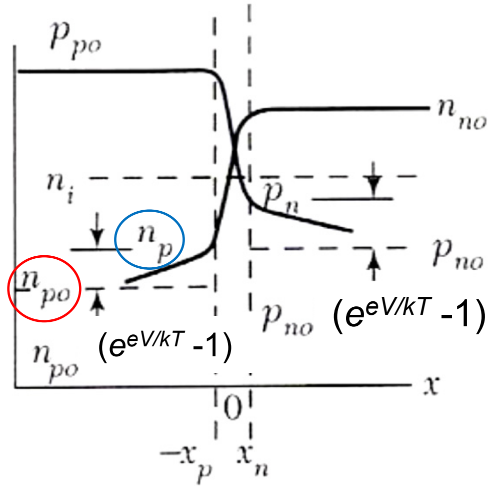
$$\frac{dn}{dt} = \frac{1}{e}\frac{dJ_n}{dx} + (G_n - R_n)$$

$$J_n = e(\mu_n nE + D_n\, dn/dx)$$

$$\frac{dn}{dt} = 0 = D_n \frac{d^2 n_p}{dx^2} + (G_n - R_n)$$

$$= D_n \frac{d^2 n_p}{dx^2} - \frac{(n_p - n_{p_0})}{\tau_n}$$

$$\frac{d^2 n_p}{dx^2} - \left(\frac{n_p - n_{p0}}{D_n \tau_n}\right) = 0$$



I-V characteristic

$$\frac{d^2 n_p}{dx^2} - \left(\frac{n_p - n_{p0}}{D_n \tau_n} \right) = 0$$

Integrate with $n_p = n_{p0}\exp(eV/k_B T)$ at $x = -x_p$ and $n_p = n_{p0}$ at $x = -\infty$ Importance of boundary conditions!

$\Rightarrow n_p - n_{p0} = n_{p0}[\exp(eV/k_B T) - 1] \exp[(x+x_p)/L_n]$ with $L_n = \sqrt{D_n \tau_n}$ ⚠ $n_p = n_p(x)$

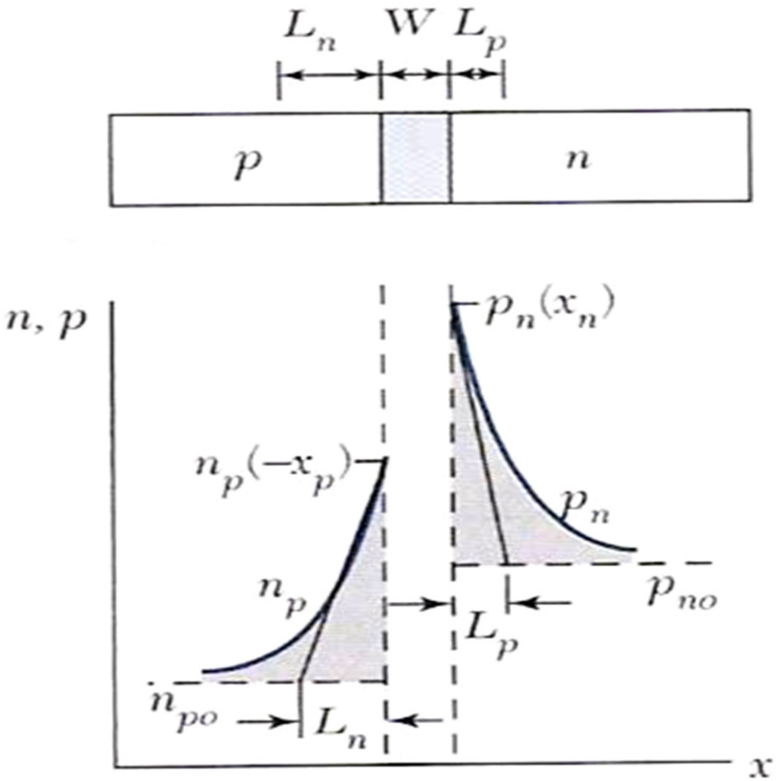
Shockley's relation, cf. slide #15

L_n is the diffusion length of minority electrons in the p-type doped layer

The electron current density at coordinate $-x_p$ is then given by $J_n(-x_p) = eD_n \left.\frac{dn_p}{dx}\right|_{-x_p} = e\frac{D_n n_{p0}}{L_n}\left(e^{eV/k_B T} - 1\right)$

and similarly the hole current density at coordinate x_n is equal to $J_p(x_n) = -eD_p \left.\frac{dp_n}{dx}\right|_{x_n} = e\frac{D_p p_{n0}}{L_p}\left(e^{eV/k_B T} - 1\right)$

I-*V* characteristic



The minority carriers recombine with the majority carriers while going away from the space charge region

Ideal *I*-*V* characteristic

The total current is:

$$J = J_n(-x_p) + J_p(x_n)$$

$$= \frac{eD_n n_{p0}}{L_n}(e^{eV/k_BT} - 1) + \frac{eD_p p_{n0}}{L_p}(e^{eV/k_BT} - 1)$$

$$J = J_s(e^{eV/k_BT} - 1) \quad \textit{with} \quad J_s = \frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}$$

Saturation current density

Use of mass action law

$n_{p0}=n_i^2/N_A$ and $p_{n0}=n_i^2/N_D$

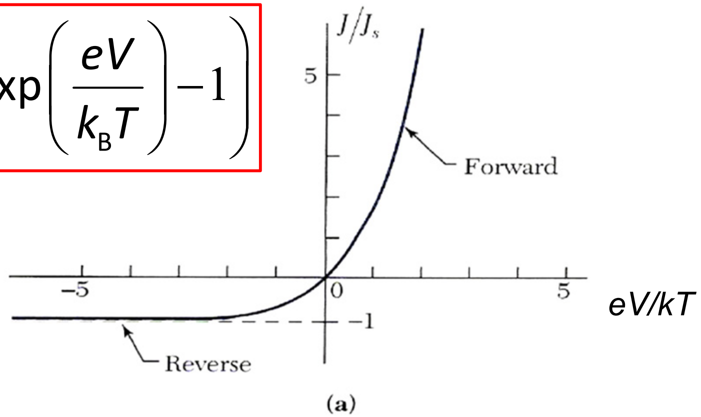
$$J_s = \frac{eD_n n_i^2}{N_A L_n} + \frac{eD_p n_i^2}{N_D L_p}$$

with $n_i^2 = N_c N_v e^{-\left(\frac{E_c - E_v}{k_BT}\right)} = N_c N_v e^{-\left(\frac{E_g}{k_BT}\right)}$

- For $V \geq 3k_BT/e$, the rate of current increase is constant
- At 300 K, for every decade change of current, the voltage change for an ideal diode is 60 mV (= $2.3k_BT/e$)
  Cf. remark of slide #8 of Lecture 8
- In the reverse direction, the current density saturates at $-J_s$

Ideal *I*-*V* characteristic

$$J(V) = J_S\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$



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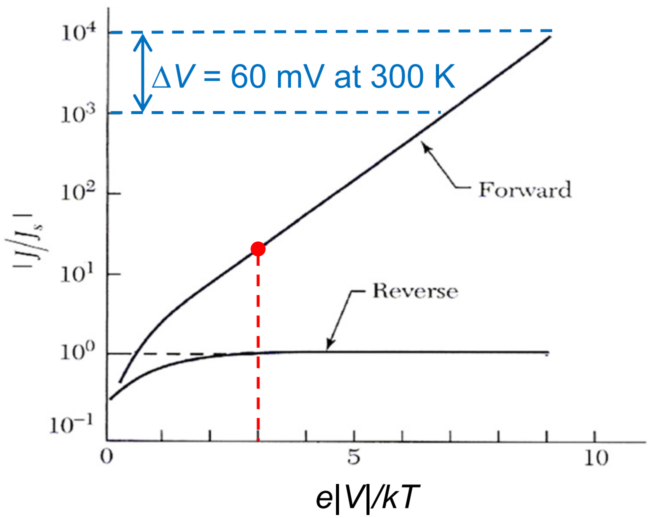

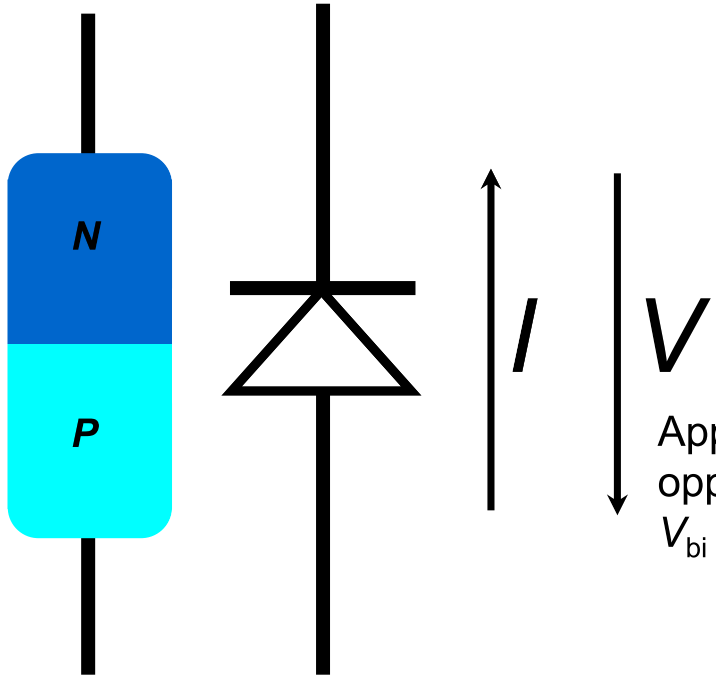

Applied bias opposite to V_{bi} for $V > 0$

$V = 0$, no current (equilibrium)

$V < 0$, J tends to – J_s (hence the denomination ***saturation current density***)

$V > 0$, J rapidly increases

Ideal *I*-*V* characteristic

J_s in a silicon-based *p*-*n* junction

$N_A = 5 \times 10^{16}$ cm^{-3}, $N_D = 1 \times 10^{16}$ cm^{-3}, $n_i \approx 10^{10}$ cm^{-3}
$D_n = 21$ cm²/s, $D_p = 10$ cm²/s, $\tau_n = \tau_p = 5 \times 10^{-7}$ s

$J_s = 8.6 \times 10$ pA/cm^2

The typical size of a device is a few squared microns in cross section, which leads to I_s values on the order of a few fA

$\times 10^{12}$

There is "no current" at reverse bias

In contrast, at $V = 1$ V (forward bias), $J = 2 \times 10^2$ A/cm^2